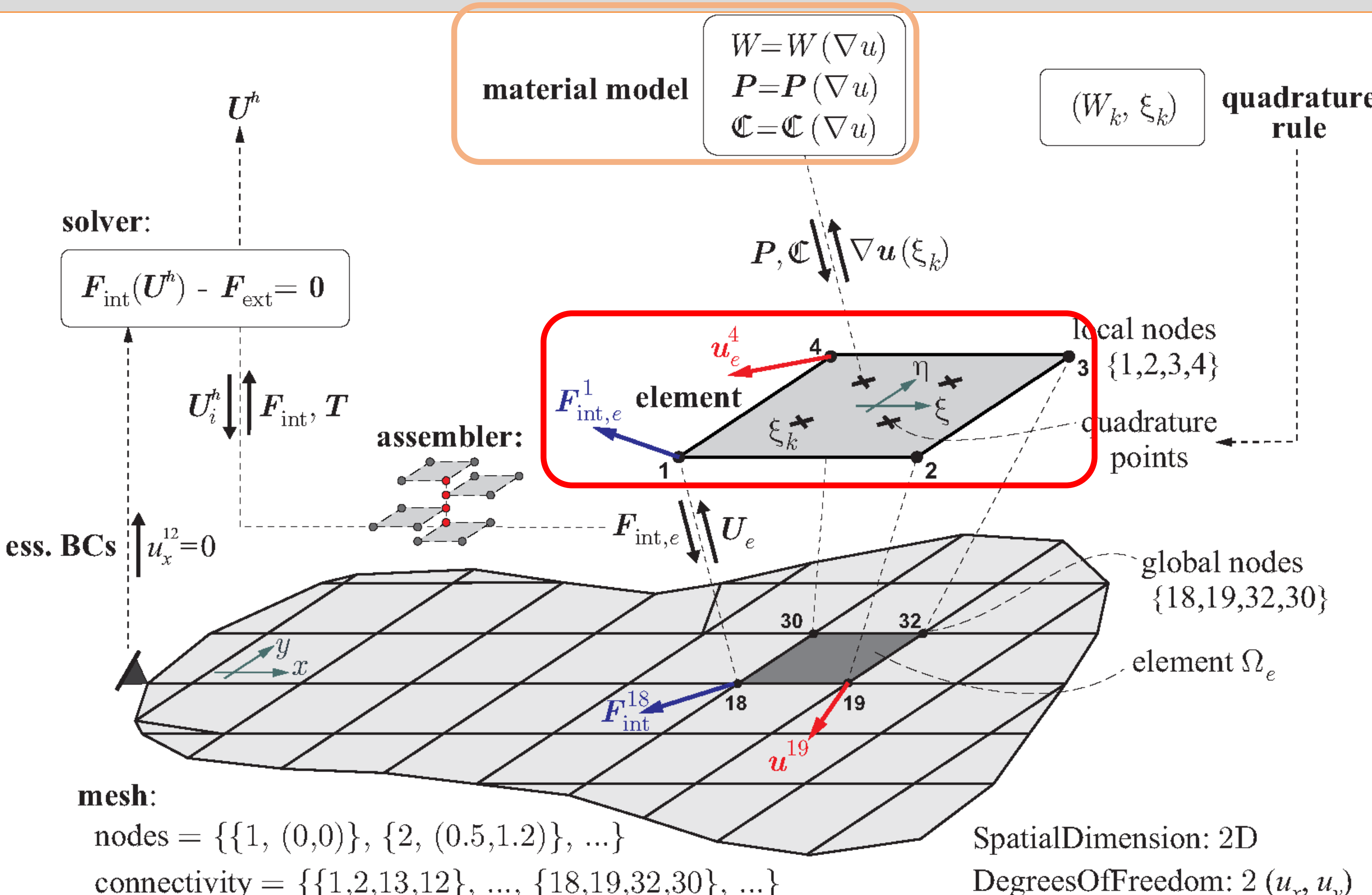
Finite Element Method: Code Overview

Ae214a: Computational Solid Mechanics



General Element Classes

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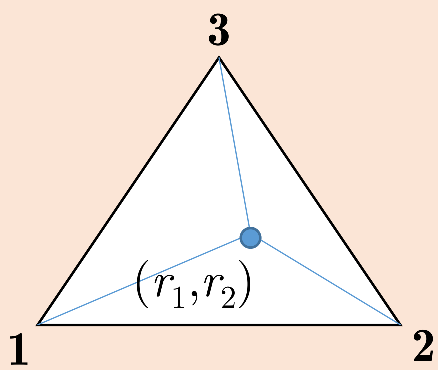
ElementType class:



element topology (basic definitions):

# of nodes in d dimension: $n = d+1$

shape functions (reference configuration):

$$N_e^1(\boldsymbol{\xi}) = r_1, \quad N_e^2(\boldsymbol{\xi}) = r_2, \ldots$$

shape function derivatives:

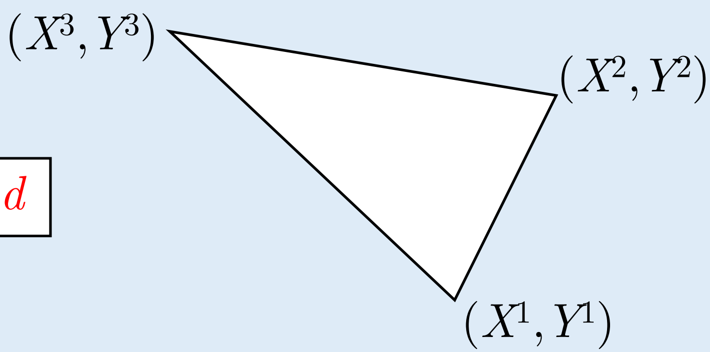
$$\nabla_{\boldsymbol{\xi}} N_e^1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad \nabla_{\boldsymbol{\xi}} N_e^n = \begin{pmatrix} -1 \\ -1 \\ \vdots \\ -1 \end{pmatrix}$$

dimension d

$\nabla_{\boldsymbol{\xi}} N_e^a(\boldsymbol{\xi}_k)$

$\boldsymbol{\xi}_k$

Element class:



element geometry: (X^i, Y^i)

use to compute the following:

$$\boldsymbol{J}(\boldsymbol{\xi}_k) = \sum_{a=1}^{n} \nabla_{\boldsymbol{\xi}} N_e^a(\boldsymbol{\xi}_k) \otimes \boldsymbol{X}^a$$

$$J(\boldsymbol{\xi}_k) = \det \boldsymbol{J}(\boldsymbol{\xi}_k)$$

$$\nabla_{\boldsymbol{X}} N_e^a(\boldsymbol{\xi}_k) = \boldsymbol{J}^{-1}(\boldsymbol{\xi}_k) \nabla_{\boldsymbol{\xi}} N_e^a(\boldsymbol{\xi}_k)$$

and **store** within the element:

$$J(\boldsymbol{\xi}_k), \quad \nabla_{\boldsymbol{X}} N_e^a(\boldsymbol{\xi}_k)$$

General Element Classes

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an element (of **any** ElementType) must compute (at least) the following quantities:

- **element energy**: $$\mathcal{I}^e = \sum_{k=1}^{n_{QP}} W_k \, J(\boldsymbol{\xi}_k) \, W\left(\nabla \boldsymbol{u}(\boldsymbol{\xi}_k)\right) t$$
- **nodal forces**: $$F^a_{\text{int},i} = \sum_{k=1}^{n_{QP}} W_k \, J(\boldsymbol{\xi}_k) \, P_{iJ}\left(\nabla \boldsymbol{u}(\boldsymbol{\xi}_k)\right) N^a_{,J}(\boldsymbol{\xi}_k) \, t$$
- **stiffness matrix**: $$T^{ab}_{ij} = \sum_{k=1}^{n_{QP}} W_k \, J(\boldsymbol{\xi}_k) \, \mathbb{C}_{iJkL}\left(\nabla \boldsymbol{u}(\boldsymbol{\xi}_k)\right) N^a_{,J}(\boldsymbol{\xi}_k) \, N^b_{,L}(\boldsymbol{\xi}_k) \, t$$

from the **Properties** (t)

from the **MaterialModel** (W, P_{iJ}, $\mathbb{C}_{iJkL}$)

from the **QuadratureRule**:
$$\{W_1, \ldots, W_{n_{QP}}\}$$
$$\{\boldsymbol{\xi}_1, \ldots, \boldsymbol{\xi}_{n_{QP}}\}$$

computed/stored by the **element constructor** ($J(\boldsymbol{\xi}_k)$, $N^a_{,J}(\boldsymbol{\xi}_k)$, $\nabla_{\boldsymbol{X}} N^a_e(\boldsymbol{\xi}_k)$)

- **displacement gradient:** $$\nabla \boldsymbol{u} = \nabla_{\boldsymbol{X}} \boldsymbol{u} = \sum_{a=1}^{n} \boldsymbol{u}^a_e \otimes \nabla_{\boldsymbol{X}} N^a_e(\boldsymbol{\xi}_k)$$

($\boldsymbol{u}^a_e$ passed to) **computeEnergy(...)**, **computeForces(...)**

Homework 5

We will look at two examples:

- **Indentation in 2D using linear elastic triangles**
- **Indentation in 3D using Neo-Hookean tetrahedra**

Note:

- Problem set #5 will be due on **Nov 23** (Wednesday before Thanksgiving).
- There will **no class** next Tuesday, **Nov 15**.